

Exam 1**Transmission Line Equations**

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$V_0^- = \Gamma V_0^+$$

$$\gamma = \alpha + j\beta$$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$v(z, t) = \Re \left\{ \tilde{V}(z) e^{j\omega t} \right\}$$

$$v(z, t) = |V_0^+| \cos(\omega t - \beta z + \phi_+) + |V_0^-| \cos(\omega t + \beta z + \phi_-)$$

$$\beta = \frac{2\pi}{\lambda} \quad (1)$$

$$\omega = 2\pi f \quad (2)$$

$$u_p = \lambda f = \frac{\omega}{\beta} \quad (3)$$

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$z_L = \zeta = \frac{Z_L}{Z_0}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} = \frac{V_0^-}{V_0^+} = -\frac{I_0^-}{I_0^+}$$

$$\Gamma = |\Gamma| e^{j\theta_r}$$

$$\begin{aligned} Z(d) &= Z_{in} = Z_d = \frac{\tilde{V}(d)}{\tilde{I}(d)} \\ &= Z_0 \left[\frac{1 + \Gamma e^{-j2\beta d}}{1 - \Gamma e^{-j2\beta d}} \right] \\ &= Z_0 \left[\frac{1 + \Gamma_d}{1 - \Gamma_d} \right] \\ &= Z_0 \left(\frac{\frac{Z_L}{Z_0} + j \tan(\beta d)}{1 + j \frac{Z_L}{Z_0} \tan(\beta d)} \right) \end{aligned}$$

$$S = \frac{|\tilde{V}|_{max}}{|\tilde{V}|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$P_{av} = \frac{|V_0^+|^2}{2Z_0} (1 - |\Gamma|^2)$$

Table 2-4 Properties of standing waves on a lossless transmission line.

Voltage maximum	$ \tilde{V} _{\max} = V_0^+ [1 + \Gamma]$
Voltage minimum	$ \tilde{V} _{\min} = V_0^+ [1 - \Gamma]$
Positions of voltage maxima (also positions of current minima)	$d_{\max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}, \quad n = 0, 1, 2, \dots$
Position of first maximum (also position of first current minimum)	$d_{\max} = \begin{cases} \frac{\theta_r \lambda}{4\pi}, & \text{if } 0 \leq \theta_r \leq \pi \\ \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2}, & \text{if } -\pi \leq \theta_r \leq 0 \end{cases}$
Positions of voltage minima (also positions of current maxima)	$d_{\min} = \frac{\theta_r \lambda}{4\pi} + \frac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \dots$
Position of first minimum (also position of first current maximum)	$d_{\min} = \frac{\lambda}{4} \left(1 + \frac{\theta_r}{\pi} \right)$
Input impedance	$Z_{\text{in}} = Z_0 \left(\frac{z_L + j \tan \beta l}{1 + j z_L \tan \beta l} \right) = Z_0 \left(\frac{1 + \Gamma_l}{1 - \Gamma_l} \right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{\text{in}} = Z_0 \left(\frac{1 + \Gamma }{1 - \Gamma } \right)$
Z_{in} at voltage minima	$Z_{\text{in}} = Z_0 \left(\frac{1 - \Gamma }{1 + \Gamma } \right)$
Z_{in} of short-circuited line	$Z_{\text{in}}^{\text{sc}} = j Z_0 \tan \beta l$
Z_{in} of open-circuited line	$Z_{\text{in}}^{\text{oc}} = -j Z_0 \cot \beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{\text{in}} = Z_L, \quad n = 0, 1, 2, \dots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$	$Z_{\text{in}} = Z_0^2 / Z_L, \quad n = 0, 1, 2, \dots$
Z_{in} of matched line	$Z_{\text{in}} = Z_0$
$ V_0^+ $ = amplitude of incident wave; $\Gamma = \Gamma e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians; $\Gamma_l = \Gamma e^{-j2\beta l}$.	

Exam 1

Electrostatics Equations

$$\begin{aligned}\nabla A &= \hat{x} \frac{\partial A}{\partial x} + \hat{y} \frac{\partial A}{\partial y} + \hat{z} \frac{\partial A}{\partial z} \\ \nabla \cdot A &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\ \nabla \times B &= \hat{x} \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) + \hat{y} \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right) + \hat{z} \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \\ \epsilon &= \epsilon_r \epsilon_0\end{aligned}$$

$$\begin{aligned}\vec{\nabla} \cdot \vec{D} &= \rho_v \\ \oint_S \vec{D} \cdot d\vec{s} &= Q \\ \vec{\nabla} \times \vec{E} &= 0 \\ \oint_C \vec{E} \cdot d\vec{l} &= 0\end{aligned}$$

$$\begin{aligned}D_{1n} - D_{2n} &= \rho_s \\ \epsilon_1 E_{1n} - \epsilon E_{2n} &= \rho_s\end{aligned}$$

$$\begin{aligned}\nabla^2 V &= \nabla \cdot (\nabla V) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\epsilon} \\ E &= -\nabla V \\ V &= -\int \vec{E} \cdot d\vec{l} \\ C &= \frac{Q}{V}\end{aligned}$$

Exam 1

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $ \mathbf{A} =$	$\sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1,$ for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1,$ for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1,$ for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}} \cdot \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A} \cdot \mathbf{B} =$	$A_x B_x + A_y B_y + A_z B_z$	$A_r B_r + A_\phi B_\phi + A_z B_z$	$A_R B_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A} \times \mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}} dx + \hat{\mathbf{y}} dy + \hat{\mathbf{z}} dz$	$\hat{\mathbf{r}} dr + \hat{\boldsymbol{\phi}} r d\phi + \hat{\mathbf{z}} dz$	$\hat{\mathbf{R}} dR + \hat{\boldsymbol{\theta}} R d\theta + \hat{\boldsymbol{\phi}} R \sin \theta d\phi$
Differential surface areas	$ds_x = \hat{\mathbf{x}} dy dz$ $ds_y = \hat{\mathbf{y}} dx dz$ $ds_z = \hat{\mathbf{z}} dx dy$	$ds_r = \hat{\mathbf{r}} r d\phi dz$ $ds_\phi = \hat{\boldsymbol{\phi}} dr dz$ $ds_z = \hat{\mathbf{z}} r dr d\phi$	$ds_R = \hat{\mathbf{R}} R^2 \sin \theta d\theta d\phi$ $ds_\theta = \hat{\boldsymbol{\theta}} R \sin \theta dR d\phi$ $ds_\phi = \hat{\boldsymbol{\phi}} R dR d\theta$
Differential volume $d\mathbf{V} =$	$dx dy dz$	$r dr d\phi dz$	$R^2 \sin \theta dR d\theta d\phi$
Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x \cos \phi + A_y \sin \phi$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r \cos \phi$ $y = r \sin \phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}} \cos \phi - \hat{\boldsymbol{\phi}} \sin \phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}} \sin \phi + \hat{\boldsymbol{\phi}} \cos \phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r \cos \phi - A_\phi \sin \phi$ $A_y = A_r \sin \phi + A_\phi \cos \phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}} \sin \theta \cos \phi + \hat{\mathbf{y}} \sin \theta \sin \phi + \hat{\mathbf{z}} \cos \theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}} \cos \theta \cos \phi + \hat{\mathbf{y}} \cos \theta \sin \phi - \hat{\mathbf{z}} \sin \theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi$	$A_R = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta$ $A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$
Spherical to Cartesian	$x = R \sin \theta \cos \phi$ $y = R \sin \theta \sin \phi$ $z = R \cos \theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}} \sin \theta \cos \phi + \hat{\boldsymbol{\theta}} \cos \theta \cos \phi - \hat{\boldsymbol{\phi}} \sin \phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}} \sin \theta \sin \phi + \hat{\boldsymbol{\theta}} \cos \theta \sin \phi + \hat{\boldsymbol{\phi}} \cos \phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos \theta - \hat{\boldsymbol{\theta}} \sin \theta$	$A_x = A_R \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$ $A_y = A_R \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$
Cylindrical to spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}} \sin \theta + \hat{\mathbf{z}} \cos \theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}} \cos \theta - \hat{\mathbf{z}} \sin \theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r \sin \theta + A_z \cos \theta$ $A_\theta = A_r \cos \theta - A_z \sin \theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R \sin \theta$ $\phi = \phi$ $z = R \cos \theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}} \sin \theta + \hat{\boldsymbol{\theta}} \cos \theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos \theta - \hat{\boldsymbol{\theta}} \sin \theta$	$A_r = A_R \sin \theta + A_\theta \cos \theta$ $A_\phi = A_\phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$